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COLLECTIVE REPUTATION AND THE DYNAMICS OF STATISTICAL DISCRIMINATION*

BY YOUNG-CHUL KIM AND GLENN C. LOURY[1]

Sogang University, South Korea; Brown University, U.S.A.

Economists have developed theoretical models identifying self-fulfilling expectations as an important source of statistical discrimination practices. The static models dominating the literature, however, may leave the false impression that a bad equilibrium is as fragile as a "bubble" and can burst at any moment when expectations flip. By developing a dynamic version of the model, we clarify the limits of expectations-related fragility. Even if group members can coordinate their expectations about future employer behavior, a group with a poor initial collective reputation may still be unable to recover its reputation, implying that the once-developed discriminatory outcomes can be long-standing.

1. INTRODUCTION

Statistical discrimination is a theory of inequality among demographic groups, where discrimination is based on stereotypes that do not arise from prejudice. When rational, information-seeking decision makers use aggregate group characteristics to evaluate relevant personal characteristics of individuals with whom they interact, individuals from different groups may be treated differently, even if they share identical observable characteristics (Moro, 2009). Economists have developed theoretical models identifying self-fulfilling expectations as an important source of statistical discrimination as practiced in labor markets.

Most notably, Arrow (1973) identified the ingredients for the existence of multiple equilibria, in which principals have different self-confirming beliefs about different social groups with identical fundamentals. Coate and Loury (1993) extend Arrow's approach and present a theory of discrimination in job assignment, where two ex ante identical groups may end up in different, Pareto-ranked, skill investment equilibria. As in Arrow's model, discrimination is generated by coordination failure: The disadvantaged group fails to coordinate on the good equilibrium. When employers expect (correctly) that fewer workers in some visibly distinct group invest in human capital, employers use more stringent hiring standards that, in equilibrium, provide lower incentives to invest in human capital, fulfilling the original expectation. Thus, when groups coordinate on different equilibria, the model displays inequality across ex ante identical groups.

A rich literature followed these early contributions based on the standard statistical discrimination frameworks, including a general equilibrium model with endogenous wages (Moro and Norman, 2004), an analysis of an economy with public and private sectors (Fang and Norman, 2006), an examination of the possibility of belief flipping in the promotion stage (Fryer, 2007), and a study of social integration and negative stereotypes (Chaudhuri and Sethi, 2008).

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The static models that dominate the literature, however, produce the unintended impression that group inequality would be fully eliminated if the disadvantaged group and firms could somehow revise their beliefs and expectations and coordinate on the good equilibrium. In other words, the models suggest that a simple change in expectations can trigger a sudden shift in behavior of employers and the disadvantaged group members, leading to equality. The models may even produce the impression that a bad equilibrium is as fragile as a "bubble" and can burst at any moment when expectations flip. Finally, and most importantly, this misperception may understate the adversity a disadvantaged group faces in escaping a bad equilibrium.[2]

Static models, by their very nature, do not explicitly consider how the average investment level of a group might evolve over time and, consequently, cannot explain under what conditions stereotypes leading to permanent inequality might arise. So, the policy implications of static models are somewhat limited.

Seeking to overcome these shortcomings, we suggest a dynamic version of the statistical discrimination model based on Coate and Loury's original setup. Dynamics are added to this setup by assuming, in a continuous time setup, that older workers retire and are replaced with new workers at the same rate. These new members—maximizing the total discounted value of future payoffs—are forward-looking. Their investment decisions are based on how they anticipate their group will be treated by employers in the future. As new members are born and older members disappear, employers' beliefs about a group's average productivity are revised over time, leading to cross-generational linkages in agents' incentives to invest. While the reputational equilibria of our dynamic model essentially replicate the equilibria of static models, the process of revision just described provides insights regarding the feasibility of the dynamic reputation recovery path from the low reputation equilibrium and the multiple dynamic paths that lead to different equilibria when a disadvantaged group's average investment level is out of equilibrium.

Through the dynamic analysis, we identify the "reputation trap" range, whereby a group is trapped with its low initial average productivity and cannot recover its reputation. The group's current newborns may not find that the conjectured future benefits can compensate for the necessary human capital acquisition cost, because it would take "too much" time for the collective reputation to improve to a level at which employers shift their treatment of the group. This formalizes the intuition that discriminatory outcomes may be persistent and that discriminated groups may be trapped by the negative influence of reputational externalities. That is, if the initial collective reputation of a disadvantaged group is too unfavorable—namely, within the reputation trap—then the group members may be unable to improve their economic welfare, even if they are able to coordinate their expectations about future employer behavior.

Beyond the reputation trap range, however, there may exist both the dynamic recovery path to the high reputation equilibrium and the dynamic deteriorating path to the low reputation equilibrium. The final state is then determined solely by whether optimistic or pessimistic expectations are formed regarding the future behavior of employers.

Thus, our analysis clarifies the limits of the above-noted expectations-related fragility, providing conditions for the persistence of statistical discrimination. A disadvantaged group with its low average productivity cannot escape the bad equilibrium, regardless of how expectations are formed. The once-developed economic disparities between demographic groups can be long-standing in the absence of some well-concerted external intervention.

This distinction between the static versus dynamic statistical discrimination models also requires a different view regarding effective government intervention. From the perspective of the static model, between-group inequality can be solved simply by the new expectation-coordination of equilibrium outcomes. Policy makers may introduce egalitarian policies with the prospect that such policies could facilitate optimistic (instead of pessimistic) expectations about the future in a targeted group, consequently leading to significant behavioral changes of group members.

[2] We appreciate an anonymous referee for articulating the weakness of the static models.

From the perspective of the dynamic model, however, the impact of such egalitarian policies could be limited. Even though the group's members are encouraged by government support and begin to seek optimistic expectations, the forward-looking economic agents may not overturn their skill acquisition behaviors if they still find that the anticipated date at which the employers' treatment of the group will shift is too distant in the future. Thus, when the dynamic aspects of statistical discrimination are not properly incorporated, a government intervention for equality could be futile, unlike the policy makers' original intentions. The policy makers who anticipated that the inequalities would disappear over time by the encouraged optimism might be embarrassed with facing only minor improvements in terms of the human capital investment activities of the targeted group.

The reminder of the article is organized as follows. In the next section, we review the relevant literature and explain its relationship to our dynamic model. In Section 3, we propose a dynamic setup of statistical discrimination, which is an extension of the static statistical discrimination model suggested by Coate and Loury (1993). In Section 4, we define the reputation trap and examine the feasibility of a reputation recovery path from each level of an initial collective reputation. Section 5 follows with a discussion of the policy interventions for equality from a dynamic perspective. Section 6 presents the conclusions of the study.

2. RELEVANT LITERATURE

In contrast to the rich literature on static models of statistical discrimination developed in the past few decades, the literature on the dynamic evolution of discrimination is sparse. As a result, our understanding of the evolution of stereotypes remains relatively poor, as summarized in Fang and Moro (2011). Nevertheless, there have been several important developments in recent years that are worthy of discussion in relation to our dynamic model.

Blume (2006) adds learning dynamics to a simplified version of Coate and Loury's model of statistical discrimination. Firms' beliefs regarding the productivity of a group are revised based on firms' collective experiences with the previous cohort, and workers' beliefs about labor market conditions are similarly updated. Presenting an asymptotic analysis of the long-run behavior of employment outcomes and agents' beliefs, Blume identifies which equilibria are stochastically stable to small randomness, given multiple static equilibria.

Antonovics (2006) presents a dynamic model of statistical discrimination that accounts for intergenerational income mobility. Statistical discrimination practices lower a disadvantaged group's average wage. The parents from the group will have fewer financial resources to invest in their children's human capital. As a result, firms will rationally discriminate against members of the group in future generations. Although her approach and ours both utilize an overlapping-generations framework to prove the existence of a low human capital investment trap, the underlying forces differ in that the trap in her model originates from the parents' limited ability to invest in human capital, whereas in ours, it arises from the negative influence of the collective reputation of previous cohorts.

Finally, Tirole (1996) studies the persistence of a poor collective reputation. His approach and ours have many similarities: Group members make binary decisions and firms' hiring decisions are partly based on their beliefs about the average behavior of the group. His work, however, is not about human capital investment activities and consequent discriminatory practices in the labor market, but more about societal phenomena, such as the prevalence of hard work and corruption. Another difference is that, in our model, for certain initial states, groups can converge to either the high-reputation state or the low-reputation state, depending on whether group members have an optimistic or a pessimistic view of the future. That is, even when the more efficient steady state is attainable, groups can become trapped if their members' collective expectations about the future remain pessimistic. The importance of coordinated expectations is widely neglected both in Tirole (1996) and in Levin's (2009) subsequent stochastic framework, which allows the cost of effort to evolve randomly, following driftless Brownian motion.

In the development of the dynamics, we are indebted to Krugman's (1991) insight regarding the interpretation of multiple self-confirming equilibrium paths. He denoted the range of multiple equilibrium paths by *overlap* in his influential argument for the relative importance of history and expectations in the determination of final economic outcomes. Within an overlap, the final state is determined by expectations about the future, whereas outside the overlap, it is determined by history. We incorporate his general arguments into the dynamic structure of statistical discrimination. Unlike his model, however, which assumes a fixed population, our dynamic model is developed in an overlapping-generations framework. We emphasize the importance of belief coordination over a long-term horizon: Expectations coordinated across different time cohorts impact the dynamic self-confirming path to be taken.

3. A DYNAMIC SETUP OF STATISTICAL DISCRIMINATION

In this section, we propose a dynamic setup of the job assignment model introduced by Coate and Loury (1993), in which worker expectations interact with employer beliefs generating multiple dynamic equilibrium paths. To illustrate effectively how a group's collective reputation evolves over time, we present the simplest possible fixed-wage market equilibrium model.

3.1. *A Static Setup of Statistical Discrimination.* First, we identify equilibrium group reputations in a static setting. Imagine a large number of identical employers and a larger population of workers, with each employer matched to many workers from this population. Employers assign each worker to one of two jobs, referred to as Task One and Task Zero. Task One is the more demanding and rewarding assignment: Workers receive a gross benefit of W if assigned to Task One. All workers prefer to be assigned to Task One, whether they are qualified for the task. Employers gain a net return of X_q if they assign a qualified worker to Task One and suffer a net loss of X_u if they assign an unqualified worker to Task One. For the sake of simplicity, a worker's gross benefit and an employer's net return from an assignment to Task Zero are normalized to be zero: The less demanding task with an insignificant reward is performed equally well by qualified and unqualified agents.

Only those who make some ex ante skill investment are qualified for Task One. The cost of obtaining a skill varies among the workers. Let c be a worker's skill investment cost and let $G(c)$ be the fraction of workers with an investment cost no greater than c.

Employers are unable to observe (prior to assignment) whether a worker is qualified for Task One. Instead, there exists a test of qualification that yields one of three outcomes: H, M, and L. The test outcome H (L) is achieved only by those who are qualified (unqualified). The test outcome M can be achieved by either those who are qualified or those who are unqualified. Let p_q (p_u) be the probability that if a worker does (does not) invest, his test outcome is M: $p_q \equiv Prob[M|\text{qualified}]$ and $p_u \equiv Prob[M|\text{unqualified}]$. We assume that the ambiguous outcome of the test, M, is more likely to be achieved if the worker is qualified: $p_q > p_u$. That is, the qualification test is more effective in identifying the unqualified instead of the qualified.

In this environment, employers' assignment decisions depend entirely on both the group identity and the test outcome. They will assign all who achieve H to the more valued task (Task One) and all who achieve L to the less valued task (Task Zero). We also know that the employer accords the "benefit of the doubt" (BOD) to a worker who achieves M by assigning him or her to Task One if and only if the expected net return from doing so is nonnegative:[3]

$$X_q \cdot Prob[\text{qualified}|M] - X_u \cdot Prob[\text{unqualified}|M] \geq 0. \tag{1}$$

[3] Employers may be indifferent regarding whether to accord the BOD or not when the expected net return from doing so is exactly zero. However, we ignore this peculiar possibility to make our analysis as simple as possible.

Let π denote the employers' belief about the proportion of qualified workers among group members. Using Bayes's rule, the posterior probability that the worker with the unclear test outcome M is qualified is $\frac{\pi p_q}{\pi p_q + (1-\pi) p_u}$ $(= Prob[\text{qualified}|M])$. Hence, the above inequality is equivalent to the requirement that

$$\pi \geq \frac{p_u X_u}{p_q X_q + p_u X_u} (\equiv \Pi^*). \quad (2)$$

Thus, a worker receives the BOD if and only if the employer is sufficiently optimistic about the rate of human capital acquisition in the population from which the worker was drawn.

Likewise, in this environment, a worker will acquire human capital if and only if the worker's cost of human capital acquisition is not greater than the anticipated return. Let $a \in [0, 1]$ denote the probability that a worker will receive the BOD in the event that this worker presents the ambiguous test outcome M. When the BOD is given, the expected net return from investing in human capital is $W(1 - p_u)$ and $W(1 - p_q)$ otherwise, given a wage rate of W for Task One. Then the expected return from investing in human capital for this worker (B) is summarized by a function of a:

$$B(a) = W[1 - p_q + a(p_q - p_u)]. \quad (3)$$

Therefore, the net return $B(a)$ is increasing with the probability that the worker receives the BOD (a), so long as our assumption that $p_q > p_u$ holds. However, in light of our discussion on employers' behavior, the BOD is given only when the "collective reputation" of the identity group to which a worker belongs is equivalent to or exceeds some threshold level Π^*, in that $a = 1$ for $\pi \geq \Pi^*$, while $a = 0$ for $\pi < \Pi^*$.

Now, it is the complementarity between a group's reputation and its members' human capital incentives that creates the possibility of multiple self-confirming reputations. Suppose that the following inequality holds: $G(B(0)) < \Pi^* < G(B(1))$, equivalently, $G(W(1 - p_q)) < \Pi^* < G(W(1 - p_u))$. First, imagine that a group's collective reputation, that is, the employers' belief about the rate of human capital acquisition among the group's population, is below the threshold Π^*: $\pi < \Pi^*$. Then, expecting that employers will withhold the BOD ($a = 0$), the group members invest in human capital acquisition at the rate $G(B(0))$, which is below Π^*. Thus, the employers' earlier belief about the group's rate of human capital acquisition is self-confirmed. An analogous explanation can be given for the self-confirmed higher skill investment rate $G(B(1))$. Given that the employers' belief about the group's rate of human capital acquisition is above the threshold Π^* (i.e., $\pi > \Pi^*$), expecting that employers will give the BOD, workers invest in human capital acquisition at the rate of $G(B(1))$, which is above Π^*. This, in turn, rationalizes the employers' optimistic belief. The following lemma summarizes these arguments.

LEMMA 1. *In the given static setup of statistical discrimination, if the following inequality holds, then both $G(B(0))$ and $G(B(1))$ are equilibrium group reputations:*

$$G(B(0)) = G(W(1 - p_q)) < \Pi^* < G(B(1)) = G(W(1 - p_u)). \quad (4)$$

3.2. *An Extension to the Dynamic Setup.* We will now extend the static model to a dynamic version of statistical discrimination. We take time to be continuous and consider agents' behavior over the interval $t \in [0, \infty)$ from an initial condition in which the group's reputation is given as $\pi_0 \in [0, 1]$.

Workers discount future receipts at a rate $\delta > 0$. Workers also face the instantaneous probability $\lambda\, dt$ of "dying" during time interval $(t, t + dt)$, that is, the death of each worker is subject to the Poisson process with parameter λ. If a worker "dies," he is immediately replaced by a "newborn" worker who must make the human capital investment decision at the moment of birth, and cannot alter that decision subsequently. Thus, newborns either invest in human capital or not, based on whether the expected discounted net return from doing so is not less than that worker's investment cost, c.

We assume that at every instant, a worker in the labor market faces the testing technology assumed above and is assigned a task based on the outcome of the test (for instance, there may be a very high turnover rate so that individual workers must recontract with different employers at every date or, equivalently, that there is no "memory" of individual workers). This labor market assumption is necessary because we want the labor market return from investing in human capital accrued at each date to be updated every moment, reflecting the evolution of the group's reputation.

We further assume that workers newly entering the market are "mixed" with the rest of the group population. That is, employers cannot perfectly distinguish the ages of the job candidates, and "age" becomes another source of imperfect information that makes the human capital acquisition behavior of previous cohorts relevant to decisions made today.[4]

Instead of the notations W, X_q, and X_u that are used in the above static setup, we use ω, x_q, and x_u in this dynamic setup: Workers receive a gross benefit of ω per unit period if assigned to Task One, and employers gain a net return of x_q per unit period from the correct assignment and incur a net loss of x_u per unit period from incorrect assignment. Both a worker's gross benefit and an employer's net return per unit period are normalized to be zero if the worker is assigned to Task Zero.

Employers behave exactly as before, extending to any worker the BOD if and only if the reputation of a worker's group exceeds a threshold at each date. Employers at date t accord the BOD to a worker with an "unclear" test result if and only if $\pi_t \geq \pi^*$, where $\pi^* \equiv \frac{p_u x_u}{p_q x_q + p_u x_u}$. Denoting a_τ as the probability of receiving the BOD in the event of an "unclear" test result at date τ, the expected return from investing in human capital accrued at date τ is represented by $\beta(a_\tau)$: $\beta(a_\tau) = \omega[1 - p_q + a_\tau(p_q - p_u)]$, according to Equation (3).

Now, imagine that a worker newly born at date t formulates expectations concerning the employers' future behavior, which, for our purposes, can be fully described by the time-varying function a_τ: $\{a_\tau\}_t^\infty$. Then the newborn worker anticipates an expected present discounted return to human capital investment (R) as a function of $\{a_\tau\}_t^\infty$:

$$R(\{a_\tau\}_t^\infty) = \int_t^\infty \beta(a_\tau)\, e^{-(\delta+\lambda)(\tau-t)} \mathrm{d}\tau. \tag{5}$$

Thus, the rate of human capital acquisition among workers newly entering the market at date t is simply $G(R(\{a_\tau\}_t^\infty))$, in which we know that $a_\tau = 1$ whenever $\pi_\tau \geq \pi^*$ and $a_\tau = 0$ whenever $\pi_\tau < \pi^*$. In the discussion below, assuming that employers have correct beliefs at every date about the human capital acquisition rate within each population group, we let π_t indicate the fraction of the living population at date t who have acquired human capital. Then, we can see from the definitions that the following differential equation must obtain[5]:

$$\frac{\mathrm{d}\pi_t}{\mathrm{d}t} = \lambda \left[G\left(R\left(\{a_\tau\}_t^\infty\right)\right) - \pi_t \right]. \tag{6}$$

In this dynamic setup, we can find the two equilibrium group reputations, which are analogous to those identified in the static setup. Suppose that the following inequality holds: $G(R(\{0\}_t^\infty)) < \pi^* < G(R(\{1\}_t^\infty))$, equivalently, $G(\frac{\omega(1-p_q)}{\delta+\lambda}) < \pi^* < G(\frac{\omega(1-p_u)}{\delta+\lambda})$. First, consider a

[4] If "age" is perfectly observable, a cohort's investment decision is not affected by the group's collective reputation. Therefore, the complementarity between employers' beliefs and the cohort's investment fully determines the economic outcome. No dynamic framework may be applicable in that case.

[5] Since $\lambda\, \mathrm{d}t$ of the total population is replaced with newborn agents in a short time interval $\mathrm{d}t$, π_t evolves in this time interval in the following way: $\pi_{t+\mathrm{d}t} = \lambda\, \mathrm{d}t \cdot G(R(\{a_\tau\}_t^\infty)) + (1 - \lambda\, \mathrm{d}t)\pi_t$. Through the rearrangement of the equation, we obtain $\frac{\pi_{t+\mathrm{d}t} - \pi_t}{\mathrm{d}t} = \lambda[G(R(\{a_\tau\}_t^\infty)) - \pi_t]$.

low initial reputation $G(\frac{\omega(1-p_q)}{\delta+\lambda})$, which is below the threshold π^*, in which the group is denied the BOD today. If workers expect that employers will withhold the BOD forever (i.e., $\{a_\tau\}_t^\infty = \{0\}_t^\infty, \forall t \geq 0$), all newborn workers invest at the rate $G(R(\{0\}_t^\infty)), \forall t \geq 0$, which is equal to $G(\frac{\omega(1-p_q)}{\delta+\lambda})$. Then, the low initial reputation $G(\frac{\omega(1-p_q)}{\delta+\lambda})$ persists, and the BOD is denied forever, self-confirming the workers' pessimistic expectations. Second, imagine a high initial reputation $G(\frac{\omega(1-p_u)}{\delta+\lambda})$, which is above the threshold π^*. If workers expect that employers will give them the BOD forever (i.e., $\{a_\tau\}_t^\infty = \{1\}_t^\infty, \forall t \geq 0$), all newborn workers invest at the rate $G(R(\{1\}_t^\infty)), \forall t \geq 0$, which is equal to $G(\frac{\omega(1-p_u)}{\delta+\lambda})$. Then, the high initial reputation $G(\frac{\omega(1-p_u)}{\delta+\lambda})$ persists and the BOD is awarded forever, self-confirming the workers' optimistic expectations. The following lemma summarizes these arguments.

LEMMA 2. *In the given dynamic setup of statistical discrimination, if the following inequality holds, then both $G(R(\{0\}_t^\infty))$ and $G(R(\{1\}_t^\infty))$ are equilibrium group reputations that correspond to the workers' pessimistic and optimistic expectations regarding the employers' future behavior respectively:*

$$G(R(\{0\}_t^\infty)) = G\left(\frac{\omega(1-p_q)}{\delta+\lambda}\right) < \pi^* < G(R(\{1\}_t^\infty)) = G\left(\frac{\omega(1-p_u)}{\delta+\lambda}\right). \tag{7}$$

In the discussion below, we assume that the above lemma holds, so that both high and low equilibrium group reputations exist. Therefore, different groups in the dynamic circumstances may undertake differing equilibrium human capital investment, exhibit contrasting levels of employment in the more demanding job, and obtain different average returns on skill investment despite having identical fundamentals with respect to investment cost and information technology.

4. REPUTATION RECOVERY IN A DYNAMIC MODEL

In this section, we examine the behavior of this dynamical system. First, we determine the feasibility of a reputation recovery path for each level of a group's initial collective reputation, defining the notion of reputation trap. Second, we seek the possibility of reputation recovery from the low equilibrium group's reputation.

To conduct the analysis more effectively, we assume the simplest form of the cost distribution: $G(c) = \Pi_l, \forall c \in [0, c_m)$ and $G(c) = \Pi_h, \forall c \in [c_m, \infty]$. This step function implies that a group's population comprises only three types of agents: Π_l is the proportion of workers who always invest in human capital, $1 - \Pi_h$ is the proportion of workers who never invest in human capital, and $\Pi_h - \Pi_l$ is the proportion of workers who invest only when the anticipated return from investing in human capital is not less than c_m. Then, we assume that $\frac{\omega(1-p_q)}{\delta+\lambda} < c_m < \frac{\omega(1-p_u)}{\delta+\lambda}$, finding that $G(\frac{\omega(1-p_q)}{\delta+\lambda}) = \Pi_l$ and $G(\frac{\omega(1-p_u)}{\delta+\lambda}) = \Pi_h$. According to Lemma 2, both Π_l and Π_h are the equilibrium group reputations that correspond to the workers' pessimistic and optimistic expectations, respectively, as far as the following inequality holds: $\Pi_l < \pi^* < \Pi_h$.

4.1. *Feasibility of a Reputation Recovery Path.* Our primary question is whether a group with a low collective reputation can recover its reputation through increased skill investment. Consider an arbitrary low initial reputation, $\pi_0 \in [\Pi_l, \pi^*)$, in which the group is denied the BOD today at time $t = 0$ (note that an initial reputation even below Π_l improves naturally up to Π_l; thus, we can skip this range).

One possibility is simply that the workers believe that employers will withhold the BOD forever (i.e., $\{a_\tau\}_t^\infty = \{0\}_t^\infty, \forall t \geq 0$). Since $R(\{0\}_t^\infty) = \frac{\omega(1-p_q)}{\delta+\lambda}, \forall t \geq 0$, the newborn workers at each date t invest at the rate Π_l, and the group's collective reputation π_t converges asymptotically to the low equilibrium reputation Π_l as $t \to \infty$, according to the differential equation (6).

However, there is another possibility: If newborns expect that, while the group is being denied the BOD today, they may still be extended this benefit in the future, and if the anticipated date at which the employers' behavior will shift is not too distant in the future, then newborns may wish to acquire human capital at a higher rate than Π_l. This enhanced human capital acquisition may raise the group's reputation to a sufficient extent that employers would, indeed, change their treatment of the group at some point in the future. That is, there may exist a date $T > 0$ such that $a_\tau = 0, \forall \tau < T$ and $a_\tau = 1, \forall \tau \geq T$, and with π_t satisfying the differential equation (6) from the initial condition $\pi_0\,(< \pi^*)$, we have that $\pi_T = \pi^*$.

The key problem is to characterize the circumstances under which reputational recovery can occur in the dynamic equilibrium of the model such that newborns' optimism about the future enables a group's reputation to recover from the initial low state within a finite period of time.

Anticipating that the BOD will be given beginning at date T, the expected return to human capital acquisition for a current newborn worker at time $t = 0$ is

$$R\left(\{0\}_0^T, \{1\}_T^\infty\right) = \int_0^T \beta(0)\, e^{-(\delta+\lambda)\tau} \mathrm{d}\tau + \int_T^\infty \beta(1)\, e^{-(\delta+\lambda)\tau} \mathrm{d}\tau = \frac{\omega}{\delta+\lambda}\left(1 - p_q + (p_q - p_u)\, e^{-(\delta+\lambda)T}\right), \tag{8}$$

which implies that the return is larger when less time T is required for employers to change their treatment of the group. Current newborns will invest at the rate Π_h only when the above return is not less than c_m. If the current newborns invest, subsequent newborn cohorts sharing the same optimistic expectations will then continue to invest at the same rate Π_h because the return is even greater for them, as the anticipated date at which employers' behavior shifts will be still less distant. Therefore, the existence condition of the dynamic recovery equilibrium is simply $R(\{0\}_0^T, \{1\}_T^\infty) \geq c_m$. Rearranging this equation, we obtain the following lemma.

LEMMA 3. *The dynamic recovery equilibrium from an arbitrary low initial reputation, $\pi_0 \in [\Pi_l, \pi^*)$, exists if and only if the conjectured time T at which employers shift their treatment of the group satisfies the following condition:*

$$R\left(\{0\}_0^T, \{1\}_T^\infty\right) \geq c_m \quad \Longleftrightarrow \quad T \leq \frac{1}{\delta+\lambda} \ln\left(\frac{\omega(p_q - p_u)}{(\delta+\lambda)c_m - \omega(1-p_q)}\right). \tag{9}$$

Using the differential equation (6) that characterizes the evolution of the group's reputation, we can determine the conjectured shift time T at which employers change their treatment of the group. Imposing the constraint that $\pi_T = \pi^*$ and applying the enhanced skill investment rate Π_h, we solve the differential equation $\frac{\mathrm{d}\pi_t}{\mathrm{d}t} = \lambda[\Pi_h - \pi_t]$, and obtain that $\pi_t = \Pi_h + (\pi^* - \Pi_h)\, e^{\lambda(T-t)}$. Then, at $t = 0$, we find the relationship between π_0 and the shift time T:

$$T = \frac{1}{\lambda} \ln\left(\frac{\Pi_h - \pi_0}{\Pi_h - \pi^*}\right), \tag{10}$$

which confirms again that a shorter shift time T is required with a higher initial collective reputation π_0. Applying time T thus calculated to the above lemma, we obtain the following conclusive result.

PROPOSITION 1. *The conjectured dynamic recovery from a poor initial collective reputation* $\pi_0 \in [\Pi_l, \pi^*)$ *is feasible if and only if the initial group reputation is sufficiently high that it satisfies*

$$\pi_0 \geq \Pi_h - (\Pi_h - \pi^*) \left(\frac{\omega(p_q - p_u)}{(\delta + \lambda)c_m - \omega(1 - p_q)} \right)^{\frac{\lambda}{\delta+\lambda}} (\equiv \bar{\pi}). \tag{11}$$

In the proposition, the right-hand side of the inequality is denoted by $\bar{\pi}$, which is below the BOD standard π^* because we assume that $\frac{\omega(1-p_q)}{\delta+\lambda} < c_m < \frac{\omega(1-p_u)}{\delta+\lambda}$. Then, the proposition implies that a group with a poor initial collective reputation below some threshold $\bar{\pi}$ may be unable to recover its reputation because current newborns may not find that conjectured future benefits can compensate for the necessary human capital acquisition costs.

Now we can define the term, "reputation trap," whereby a group is trapped with its low initial collective reputation and cannot recover its reputation in the absence of external intervention. It is notable that, given that $\bar{\pi} \leq \Pi_l$, the reputation recovery path is feasible for every initial level of the collective reputation within Π_l and π^* according to Proposition 1, and thus the recovery path is feasible as well for any initial reputation even below Π_l, $\forall \pi_0 \in [0, \Pi_l)$, because the group's reputation improves naturally up to Π_l. Therefore, the reputation trap range exists only when $\bar{\pi} > \Pi_l$, and the entire reputation trap range is $[0, \bar{\pi})$ when it exists, as summarized in the following corollary:

COROLLARY 1. *If and only if* $\bar{\pi} > \Pi_l$ *does the reputation trap range exist, and the entire trap range is* $[0, \bar{\pi})$*, within which the group's reputation cannot be recovered in the absence of external intervention.*

From the definition of $\bar{\pi}$ in Proposition 1, we can also infer that when the reputation trap range exists, its size (which equals $\bar{\pi}$) is an increasing function of the time discount factor δ: $\frac{\partial \bar{\pi}}{\partial \delta} > 0$. That is, the more myopic group members are, the more likely it is that the group falls within the low reputation trap.

On the other hand, beyond the reputation trap range, both the dynamic recovery path to the high equilibrium state Π_h and the dynamic "deteriorating" path to the low equilibrium state Π_l are obtainable for the disadvantaged group even when its collective reputation is still below π^* (i.e., $\forall \pi_0 \in [\bar{\pi}, \pi^*)$). The dynamic self-confirming path to be taken depends on the expectations coordinated across different time cohorts regarding the future behavior of employers. If an optimistic view that employers will change their treatment of the group at some point in the future is shared among group members, the group's collective reputation converges to the high equilibrium reputation, while if a pessimistic view that employers will withhold the BOD forever is shared among group members, it converges to the low equilibrium reputation.

4.2. *Reputation Recovery from Low Equilibrium State* (Π_l). In the standard statistical discrimination literature, groups coordinate on different self-confirming equilibria, and thus between-group inequality can arise among the ex ante identical groups. Static models dominating the literature, however, produce the unintended impression that if the disadvantaged group members and employers could somehow revise their beliefs and expectations and coordinate on the good equilibrium, the between-group inequality would be fully eliminated.

For instance, in our static setup, two equilibrium group reputations, $G(B(0))$ and $G(B(1))$, exist, as summarized in Lemma 1. If disadvantaged group members at the bad equilibrium change expectations regarding employers' treatment in the labor market from $a = 0$ to $a = 1$, the rate of human capital acquisition among the group's population is enhanced from $G(B(0))\,(< \pi^*)$ to $G(B(1))\,(> \pi^*)$, and thus employers begin to accord the group members the BOD.

That is, the simple change in expectations can trigger a sudden shift in the behavior of the disadvantaged group members and the decision making of employers, eliminating the between-group inequality. Therefore, in the given static setup, a bad equilibrium can be considered as fragile as a "bubble" that can burst at any moment when expectations about the future flip.

In this section, we clarify the limits of this expectations-related fragility by examining the dynamic setup developed above, in which dynamics are added by assuming that older workers retire and are replaced with new workers at the same rate. As workers are replaced, employers' beliefs about a group's average productivity are revised gradually over time. As summarized in Lemma 2, we can replicate the two equilibrium group reputations in this dynamic setup: $G(R(\{0\}_t^\infty))$ and $G(R(\{1\}_t^\infty))$. Can the change in expectations eliminate the between-group inequality even in this dynamic setup?

First, acknowledge that it is not rational for the disadvantaged group members at the low equilibrium to change their expectations regarding employers' future behavior $\{a_\tau\}_0^\infty$ directly from $\{0\}_0^\infty$ to $\{1\}_0^\infty$ because it must take some time for employers' beliefs about the group's average productivity to be improved to a sufficient extent. Instead, the group members may change their expectations $\{a_\tau\}_0^\infty$ from $\{0\}_0^\infty$ to $\{0\}_0^T$ and $\{1\}_T^\infty$, in which the anticipated shift in the behavior of employers occurs at some future date T. This change of expectations and consequent enhancement of human capital acquisition activities may or may not eliminate the possible between-group inequality. As discussed earlier, if the conjectured shift time T is too distant, the newborn workers may not find that the discounted net return from the optimistic scenario of $\{0\}_0^T$ and $\{1\}_T^\infty$ still exceeds the incurred cost c_m.

From Proposition 1, we already know that a group with the low equilibrium reputation Π_l cannot escape the low skill investment trap if and only if the group's current reputation (Π_l) is below some threshold $\bar{\pi}$: $\Pi_l < \bar{\pi}$. Furthermore, according to Corollary 1, this condition is equivalent to the existence condition of the reputation trap range $[0, \bar{\pi})$. That is, whenever there exists a reputation trap range, the low equilibrium group reputation Π_l falls within this trap. Then, we obtain the following conclusive result.

THEOREM 1. *Any group with the low equilibrium reputation Π_l cannot recover its reputation even through the change of the coordinated expectations among the group members if and only if the group's overall skill level (Π_l) is below the threshold $\bar{\pi}$, equivalently that*

$$\left(\frac{\Pi_h - \Pi_l}{\Pi_h - \pi^*}\right) > \left(\frac{\omega(p_q - p_u)}{(\delta+\lambda)c_m - \omega(1-p_q)}\right)^{\frac{\lambda}{\delta+\lambda}}. \tag{12}$$

The theorem formalizes the intuition that discriminated groups at the equilibrium may be trapped by the negative influence of reputational externalities: The group cannot escape the bad equilibrium regardless of how expectations are formed, and the once-developed discriminatory outcomes can be long-standing.

Furthermore, the feasibility of the reputation recovery can be interpreted in terms of other parameters, such as time discount factor δ. Although a group's initial collective reputation Π_l may not be especially poor, the group may be trapped in the low-skill investment activities if group members weight too heavily the near future or are strongly myopic. Thus, from the above theorem, we directly obtain the following auxiliary result.

COROLLARY 2. *The conjectured dynamic recovery from the low equilibrium group reputation Π_l is not possible if and only if the group members' time discount factor δ is sufficiently great that it satisfies*

$$\frac{\delta}{\lambda} > \frac{\ln(\omega(p_q - p_u)) - \ln((\delta+\lambda)c_m - \omega(1-p_q))}{\ln(\Pi_h - \Pi_l) - \ln(\Pi_h - \pi^*)} - 1. \tag{13}$$

5. POLICY IMPLICATIONS

From the perspective of the static model, between-group inequality persists due to the continuing coordination failure: Given the equilibrium multiplicity, the disadvantaged group continues to fail to coordinate on the good equilibrium played by the dominant group. Policy makers may believe that the encouraged optimism among the disadvantaged group members can cause a significant behavioral change in terms of the group's human capital acquisition activities. In this regard, a government may implement various policy measures that could help facilitate optimistic expectations about the future.

However, this perception regarding effective policy intervention may understate a rather complicated situation that a disadvantaged group faces in escaping a bad equilibrium. Even though a government intervention is introduced with the prospect that the policy could provide encouragement for the change of expectations, its impact could be limited when the dynamic aspects of statistical discrimination are not properly taken into account in the implementation stage.

In this section, we examine two typical policy measures that a government may implement to encourage collective optimism and the subsequent behavioral changes in a disadvantaged group: (i) training subsidies and (ii) favorable hiring standards. Our goal is to confirm how limited the impact of such policies would be if the dynamics of collective reputation were not seriously considered.

In order to examine the effectiveness of a government intervention, we need to relax the strict assumption imposed on the distribution of human capital acquisition cost in the previous section. Instead of assuming the three types of cost agents, we now assume that the human capital acquisition cost is uniformly distributed between 0 and k: $G(c) = c/k$, $\forall c \in [0, k]$. Then, according to Lemma 2, both $\frac{\omega(1-p_q)}{k(\delta+\lambda)}$, denoted by π_L, and $\frac{\omega(1-p_u)}{k(\delta+\lambda)}$, denoted by π_H, are the equilibrium group reputations as far as the following inequality holds: $\pi_L < \pi^* < \pi_H$. In the following discussion, we consider a disadvantaged group at the low reputation equilibrium π_L.

5.1. *Training Subsidies.* Suppose that the government seeking to facilitate collective optimism in a targeted group introduces a training subsidy program that can reduce the human capital acquisition cost of each member of the disadvantaged group as much as S. The government continues this program until the collective reputation of the group reaches the BOD threshold π^*. We examine below whether this intervention can make the reputation recovery attainable for the disadvantaged group members from the dynamic perspectives.

In the given dynamic framework, a group with a low initial reputation π_L can recover its reputation only when the newborns can rationally expect that while the group is denied the BOD today, they may yet be extended this benefit within a finite period of time, as other group members increase the skill investment rate under the introduced subsidy program. Consider this finite period of time T such that $a_\tau = 0$ for any $\tau \in [0, T)$ and $a_\tau = 1$ for any $\tau \in [T, \infty]$. Owing to the introduced training subsidy S, the expected skill acquisition rate among the newborn cohort at a future time $t \in [0, T]$ is $G(R(\{a_\tau\}_t^\infty) + S)$ because each newborn worker will invest in human capital acquisition only when the expected return is not less than the acquisition cost minus the subsidy: $R(\{a_\tau\}_t^\infty) \geq c - S$, in which $R(\{a_\tau\}_t^\infty) = R(\{0\}_t^T, \{1\}_T^\infty)$ for any $t \in [0, T]$. This expected skill acquisition rate at a future time $t \in [0, T]$ is computed as, using Equation (5),

$$
\begin{aligned}
(14) \quad G\left(R\left(\{0\}_t^T, \{1\}_T^\infty\right) + S\right) &= G\left(\int_t^T \beta(0)\, e^{-(\delta+\lambda)(\tau-t)} \mathrm{d}\tau + \int_T^\infty \beta(1)\, e^{-(\delta+\lambda)(\tau-t)} \mathrm{d}\tau + S\right) \\
&= \frac{\omega(1-p_q)}{k(\delta+\lambda)} \cdot [1 - e^{-(\delta+\lambda)(T-t)}] + \frac{\omega(1-p_u)}{k(\delta+\lambda)} \cdot e^{-(\delta+\lambda)(T-t)} + \frac{S}{k} \\
&= e^{-(\delta+\lambda)(T-t)}(\pi_H - \pi_L) + \pi_L + \frac{S}{k}.
\end{aligned}
$$

Then, applying this to Equation (6), we can express how the fraction of the living population who have acquired human capital at time $t \in [0, T]$ evolves over time under the expectations that the BOD is given since time T:

$$\frac{d\pi_t}{dt} = \lambda[e^{-(\delta+\lambda)(T-t)}(\pi_H - \pi_L) - (\pi_t - \pi_L - S/k)],\ \forall t \in [0, T]. \tag{15}$$

This differential equation can be solved using the boundary condition that $\pi_T = \pi^*$. First, let us denote that $y_t \equiv \frac{\pi_t - \pi_L - S/k}{\pi_H - \pi_L} e^{(\delta+\lambda)(T-t)}$. Then, the above differential equation is simplified with

$$\frac{dy_t}{dt} = -(\delta + 2\lambda)y_t + \lambda,\ \text{given}\ y_T = \frac{\pi^* - \pi_L - S/k}{\pi_H - \pi_L}\ (\equiv \bar{y_T}). \tag{16}$$

The solution of this simplified differential equation is achieved using the following lemma.

LEMMA 4. *The solution of a differential equation* $\frac{dy_t}{dt} = -(\delta + 2\lambda)y_t + \lambda$ *is, given a boundary condition that* $y_T = \bar{y_T}$*:*

$$y_t = \left(\bar{y_T} - \frac{\lambda}{\delta + 2\lambda}\right) e^{(\delta+2\lambda)(T-t)} + \frac{\lambda}{\delta + 2\lambda}. \tag{17}$$

PROOF. From the differential equation, we achieve directly that $y_t = e^{-(\delta+2\lambda)t+C} + \frac{\lambda}{\delta+2\lambda}$, in which C is an arbitrary number. Applying the boundary condition to this, we obtain that $\bar{y_T} = e^{-(\delta+2\lambda)T+C} + \frac{\lambda}{\delta+2\lambda}$. Eliminating the arbitrary number C, we arrive at the solution. ■

Replacing y_t and $\bar{y_T}$ in the above lemma with $y_t \equiv \frac{\pi_t - \pi_L - S/k}{\pi_H - \pi_L} e^{(\delta+\lambda)(T-t)}$ and $\bar{y_T} \equiv \frac{\pi^* - \pi_L - S/k}{\pi_H - \pi_L}$, we achieve the immediate solution of the differential equation (15):

$$\frac{\pi_t - \pi_L - S/k}{\pi_H - \pi_L} e^{(\delta+\lambda)(T-t)} = \left(\frac{\pi^* - \pi_L - S/k}{\pi_H - \pi_L} - \frac{\lambda}{\delta + 2\lambda}\right) e^{(\delta+2\lambda)(T-t)} + \frac{\lambda}{\delta + 2\lambda}. \tag{18}$$

The rearrangement gives us the solution in terms of π_t:

$$\pi_t = \left(\pi^* - \frac{\delta + \lambda}{\delta + 2\lambda}\pi_L - \frac{\lambda}{\delta + 2\lambda}\pi_H - \frac{S}{k}\right) e^{\lambda(T-t)} + \frac{\lambda(\pi_H - \pi_L)}{\delta + 2\lambda} e^{-(\delta+\lambda)(T-t)} + \pi_L + \frac{S}{k}. \tag{19}$$

The conjectured dynamic recovery path from the low reputation equilibrium π_L will be obtainable if and only if there exists a finite period of time T that satisfies the above solution equation, given the initial condition $\pi_0 = \pi_L$. Applying $t = 0$ at Equation (19), the finite period of time T must satisfy the following equation:

$$-\pi^* + \frac{\delta + \lambda}{\delta + 2\lambda}\pi_L + \frac{\lambda}{\delta + 2\lambda}\pi_H + \frac{S}{k} = \frac{\lambda(\pi_H - \pi_L)}{\delta + 2\lambda} e^{-(\delta+2\lambda)T} + \frac{S}{k} e^{-\lambda T}. \tag{20}$$

This implies that the conjectured time $T(< \infty)$ at which employers shift their treatment of the group exists only when the left-hand side (LHS) of the above equation is positive. In other words, if the LHS of the equation is either zero or negative, a reasonable conjectured time $T(< \infty)$ that satisfies the solution equation (19) given $\pi_0 = \pi_L$ does not exist, which means that the reputation recovery is never feasible, even with the introduced government subsidy program. The above discussion is summarized by the following proposition.

PROPOSITION 2. *Even when a government subsidy program that reduces the human capital acquisition cost as much as S is introduced to facilitate the new coordination of the equilibrium outcome, the dynamic reputation recovery from the low reputation equilibrium π_L is never feasible if and only if the following inequality holds:*

$$\frac{\delta+\lambda}{\delta+2\lambda}\pi_L + \frac{\lambda}{\delta+2\lambda}\pi_H + \frac{S}{k} \le \pi^*. \tag{21}$$

Therefore, the subsidy intervention could be futile in the sense that the members of the disadvantaged group will not change their skill acquisition behaviors substantially, unlike the policy makers' original intentions, when their initial reputation at the equilibrium (π_L) is sufficiently poor that it still satisfies the condition (21).

However, this does not mean that the government program does not make a difference at all. Even when it is anticipated by the group members that employers will withhold the BOD forever (i.e., $\{a_\tau\}_0^\infty = \{0\}_0^\infty$), there should be at least a minor improvement in the skill investment rate among the newborn cohorts from $G(R(\{0\}_0^\infty))$ to $G(R(\{0\}_0^\infty) + S)$. Thus, in this case, the group's collective reputation converges over time to the "new" low reputation equilibrium, which is $\pi_L + S/k$.

One might wonder, then, how policy makers who understand the reputation trap from the dynamic perspective may react differently. Primarily, they will search for the minimal size of subsidy (S^*) that can induce the dynamic reputation recovery from the low reputation equilibrium, which can be computed in the given dynamic setup directly from the condition (21): $S^* = k[\pi^* - \pi_L - (\pi_H - \pi_L)(\delta/\lambda + 2)^{-1}]$. It is notable that this minimum threshold is affected by how much the group members weight the near future (δ) and how poor is the disadvantaged group's labor market reputation (π_L): The more myopic they are or the poorer the average productivity is, the greater is the minimum threshold S^* required. The decision whether to implement this effective subsidy program may largely depend on the political commitments and resource availability of the society in which they live.

5.2. *Favorable Hiring Standards.* A similar discussion can be applied to a government policy that induces favorable hiring standards imposed on disadvantaged group members by employers. In the given theoretical framework, the degree of such government intervention is measured by $a_t\,(> 0)$ given $\pi_t < \pi^*$, which represents some positive probability of receiving the BOD even when the group's collective reputation is below the threshold π^*. Although the government introduces this policy with the prospect that the facilitated optimistic expectations through the enhanced return to human capital investment may influence the skill acquisition behaviors of the disadvantaged group members, the actual impact of the policy could be fairly limited from the dynamic perspective, failing the new coordination of the equilibrium outcome.

The proof is almost identical with the case of training subsidy program. For the sake of simplicity, let us suppose that the degree of the government intervention is fixed as $\tilde{a} \in (0, 1)$ given $\pi_t < \pi^*$. The reputation recovery is feasible only when the newborns can rationally expect that there exists a finite period of time T such that $a_\tau = \tilde{a}$ for any $\tau \in [0, T)$ and $a_\tau = 1$ for any $\tau \in [T, \infty]$. Since $\beta(\tilde{a}) = \omega[1 - p_q + \tilde{a}(p_q - p_u)]$, the expected skill investment rate at a future time $t \in [0, T]$ is computed as

$$\begin{aligned} G\left(R\left(\{\tilde{a}\}_t^T, \{1\}_T^\infty\right)\right) &= G\left(\int_t^T \beta(\tilde{a})\, e^{-(\delta+\lambda)(\tau-t)} \mathrm{d}\tau + \int_T^\infty \beta(1)\, e^{-(\delta+\lambda)(\tau-t)} \mathrm{d}\tau\right) \\ &= \frac{\omega(1-p_q) + \omega\tilde{a}(p_q - p_u)}{k(\delta+\lambda)} \cdot [1 - e^{-(\delta+\lambda)(T-t)}] + \frac{\omega(1-p_u)}{k(\delta+\lambda)} \cdot e^{-(\delta+\lambda)(T-t)} \\ &= (\pi_L + \tilde{A}) \cdot [1 - e^{-(\delta+\lambda)(T-t)}] + \pi_H\, e^{-(\delta+\lambda)(T-t)}, \end{aligned} \tag{22}$$

in which $\frac{\omega(p_q-p_u)}{k(\delta+\lambda)}\tilde{a}$ is denoted by $\tilde{A}$. Applying this to Equation (6), we can express how the fraction of the living population who have acquired human capital (π_t) evolves over time:

$$\frac{\mathrm{d}\pi_t}{\mathrm{d}t} = \lambda[e^{-(\delta+\lambda)(T-t)}(\pi_H - \pi_L - \tilde{A}) - (\pi_t - \pi_L - \tilde{A})],\ \forall t \in [0, T]. \tag{23}$$

This differential equation is solved using the boundary condition that $\pi_T = \pi^*$. Let us denote that $z_t \equiv \frac{\pi_t-\pi_L-\tilde{A}}{\pi_H-\pi_L-\tilde{A}}\, e^{(\delta+\lambda)(T-t)}$. Then, the above differential equation is simplified with

$$\frac{\mathrm{d}z_t}{\mathrm{d}t} = -(\delta+2\lambda)z_t + \lambda,\ \text{given } z_T = \frac{\pi^* - \pi_L - \tilde{A}}{\pi_H - \pi_L - \tilde{A}}\ (\equiv \bar{z_T}). \tag{24}$$

Using Lemma 4, we can directly infer that the solution of this differential equation is

$$z_t = \left(\bar{z_T} - \frac{\lambda}{\delta+2\lambda}\right) e^{(\delta+2\lambda)(T-t)} + \frac{\lambda}{\delta+2\lambda}. \tag{25}$$

Replacing z_t and $\bar{z_T}$ in the above equation with $z_t \equiv \frac{\pi_t-\pi_L-\tilde{A}}{\pi_H-\pi_L-\tilde{A}}\, e^{(\delta+\lambda)(T-t)}$ and $\bar{z_T} \equiv \frac{\pi^*-\pi_L-\tilde{A}}{\pi_H-\pi_L-\tilde{A}}$, we obtain the solution of the differential equation (23). Through the rearrangement, we arrive at the following solution equation with respect to π_t:

$$\pi_t = \left(\pi^* - \frac{\delta+\lambda}{\delta+2\lambda}(\pi_L+\tilde{A}) - \frac{\lambda}{\delta+2\lambda}\pi_H\right) e^{\lambda(T-t)} + \frac{\lambda(\pi_H-\pi_L-\tilde{A})}{\delta+2\lambda} e^{-(\delta+\lambda)(T-t)} + \pi_L + \tilde{A}. \tag{26}$$

Applying $t = 0$ at the above equation, the finite period of time T must satisfy the following equation, given the initial condition that $\pi_0 = \pi_L$:

$$-\pi^* + \frac{\delta+\lambda}{\delta+2\lambda}(\pi_L+\tilde{A}) + \frac{\lambda}{\delta+2\lambda}\pi_H = \frac{\lambda(\pi_H-\pi_L-\tilde{A})}{\delta+2\lambda} e^{-(\delta+2\lambda)T} + \tilde{A}\, e^{-\lambda T}. \tag{27}$$

This implies that the conjectured time $T(<\infty)$ exists if and only if the LHS of the above equation is positive. Otherwise, the reputation recovery is never feasible and we cannot expect a substantial improvement of the skill acquisition rate among the group's newborn workers. The above discussion is summarized by the following proposition.

PROPOSITION 3. *Even when the government enhances the probability of the disadvantaged group members receiving the benefit of the doubt from 0 to* $\tilde{a} \in (0,1)$ *to facilitate the new coordination of the equilibrium outcome, the dynamic reputation recovery from the low reputation equilibrium* π_L *is never feasible if and only if the following inequality holds:*

$$\frac{\delta+\lambda}{\delta+2\lambda}(\pi_L+\tilde{A}) + \frac{\lambda}{\delta+2\lambda}\pi_H \le \pi^*,\ \textit{in which } \tilde{A} \equiv \frac{\omega(p_q-p_u)}{k(\delta+\lambda)}\tilde{a}. \tag{28}$$

Therefore, even though the return on human capital investment is enhanced by government intervention, we cannot anticipate the reputation recovery when the group's initial collective reputation (π_L) is sufficiently poor that it satisfies the condition (28). In such a case, there will be only a minor improvement in the human capital investment rate from $G(R(\{0\}_0^\infty))$ to $G(R(\{\tilde{a}\}_0^\infty))$. Thus, the group's reputation converges over time to the "new" low reputation equilibrium, which is $\pi_L + \tilde{A}$.

On the other hand, policy makers who are enlightened by the dynamic aspects of collective reputation will search for the minimum treatment threshold ($\tilde{a}^*$) for the effective government program that can induce the escape from the low reputation equilibrium, which is $\tilde{a}^* = k[\omega(p_q - p_u)]^{-1}[(\delta + \lambda)(\pi^* - \pi_L) - \lambda(\pi_H - \pi^*)]$ in the given dynamic setup (condition (28)). It is confirmed again that the more myopic the group members are (the greater δ is) or the poorer the group's average productivity is (the lower π_L is), the greater is the minimum threshold $\tilde{a}^*$ required.

So far, we have examined two exemplary government interventions for equality. The policy makers who only consider the static models of statistical discriminations may interpret that the between-group inequality can be solved simply by the new expectation-coordination of the equilibrium outcome. To encourage the shift in expectations, they may introduce some government interventions into either the human capital investment phase (e.g., training subsidy) or the labor market phase (e.g., favorable hiring standards). As discussed above, however, the impact of either intervention could be limited, especially when the initial skill level of the disadvantaged group is very poor. Even though the group's members are encouraged by the government support and seek optimistic (instead of pessimistic) expectations, the forward-looking newborn agents trapped by the negative influence of reputational externalities may not overturn their skill acquisition behaviors. Then, the policy makers who have anticipated the new coordination will be embarrassed with only minor improvements in terms of the group's human capital investment activities.

6. CONCLUSION

This article adds a new twist to the highly developed literature on self-fulfilling collective reputation since Arrow's (1973) seminal work. Past studies have shown that multiple equilibria may exist when a poor group reputation reduces the value of investment in human capital to group members so that the poor group reputation becomes self-fulfilling. The present article adds a dynamic analysis of a somewhat specialized model that shows how reputation can evolve from different starting points. When initial group reputations are not too extreme and if workers are sufficiently patient, then both dynamic self-confirming paths to low and high equilibrium states are obtainable: Either path can be attained, depending on the coordinated expectations of group members. If initial group reputations are too extreme (or workers too impatient), however, the group with a poor initial collective reputation will remain in a low skill investment trap and cannot escape it, regardless of how the expectations of its members are formed. Thus, the dynamic model proves that although between-group disparities may originate from a failure of expectation coordination (or a dramatic historical event), in some circumstances, the once-developed discriminatory outcomes can be long-standing.

We also note the potential pitfalls of the policy recommendation, based on the static model, that the encouragement of new expectation coordination among disadvantaged group members can eliminate between-group inequality. The government may introduce various measures to encourage collective optimism about the future. Nevertheless, the group may still remain in the low reputation trap, and the policy impact could be limited from a dynamic perspective. We suggest that a successful intervention must be planned carefully to ensure that the dynamic reputation recovery is feasible for a group trapped in low skill investment activities.

In this regard, we acknowledge that the given dynamic approach offers different perspectives in evaluating egalitarian policies. For instance, in many academic papers based on static models (e.g., Coate and Loury, 1993; Moro and Norman, 2004), the policy discussion takes the form of how to eliminate the bad equilibrium without (or possibly at the risk of) generating another bad equilibrium. However, in the given dynamic model, the discussion instead takes the form of how to effectively generate a good equilibrium path when such a path does not exist.[6]

[6] We thank an anonymous referee for helping us formulate this distinction between the static versus dynamic models.

Although we have discussed the feasibility of a recovery path in a dynamic context, we have not yet discussed how expectations about the future are formed across different time cohorts. The mechanism of belief formation is an important topic; however, it is beyond the scope of this article and thus is left for future research. Furthermore, in our dynamic model, given the assumption that wages are exogenously determined, inter-group interactions in the labor market are not considered at all. An examination on the cross-group effects in the given dynamic framework may provide a potentially fruitful avenue for future research.

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